

there is but one step between the impromptu dance or poem which tells of a recent occurrence and the work of art which transmits the memory of that occurrence to posterity, yet it appears that there are savages who have no historical art. On the other hand, the historical art has everywhere reached its highest state of development amongst nations who have had to hold their own against neighbouring tribes.

Before discussing the problems of art and sexual selection, of the origin of self-decoration and of erotic art, the author devotes a chapter to a consideration of animal display, and his treatment of the subject is worthy of the attention of zoologists. He arrives at the conclusion that human sexual selection did not create any quality of beauty and that human decoration, like that of animals, is mainly an advertisement of likeness of kind; but, strange as it may appear, scarcely any form of dress or ornament can be quoted which could be considered with certainty an outcome of the impulse to attract or charm the opposite sex. Decorations of various kinds are conferred on young people on attaining puberty, and indicate a new social status, and various subsequent advances in rank have their appropriate decoration. The impulse to ostentation with regard to rank, valour or wealth is undeniably independent of sexual selection. Even where there is no competition between rivals, sexual emotions may still find an artistic expression. Like the courting display of many birds, men may have resorted to song and dance as a mode of overcoming the instinctive coyness of the female after sexual selection has operated; but the strong emotional tension of such periods must in any case seek relief by sound or movement.

It is evident that a pantomimic imitation of any activity must, as exercise and stimulation, facilitate the subsequent real execution of the same activity. Individuals and nations who have grown familiar in play with the most important actions in life's work have thus acquired an unquestionable advantage in the struggle for existence. This holds good alike for the everyday occupations of life as for war. Music and song have especially been useful stimuli to work, partly to overcome natural laziness or inertia, partly to effect unison in the actions of several workers; for instance, the regularity of the action of many peoples is explicable as a result of the rhythmic songs by which their work is accompanied. This applies with equal force to war; hence it is not surprising to find highly developed choral dances in those peoples in whose life war is a customary occurrence. The need of stimulation is never so great as when a man has to risk his life in an open battle, and with this end in view the military singers of some tribes are able to work themselves and their audience up to a pitch of frenzy which is almost equal to that produced by the dances. Courage is also induced by the effort to appear formidable and courageous. Instruction in grimacing even formed a part of the military education of the Maoris. Hence, too, the frightful decorations which so many peoples employ when going on the warpath and the well-known face-shields of some of the tribes of New Guinea and Borneo. The decorative art of warlike peoples is usually characterised by a vigour and originality which dominate also their poetry and dramatic dances, and which are

the outcome of an intense and forcible life; but descriptive and figurative art, in the sense of realistic, faithful rendering of nature and life, has never attained any high development among the most military tribes.

Sympathetic magic which is based upon a likeness between things calls forth imitations of nature and life which, although essentially non-æsthetic in their intention, may nevertheless be of importance for the historical evolution of art. Nor is this confined to the primitive or decorative arts. There are many magical dances and pantomimes, and there is an universal belief in the efficacy of incantations and in magical songs and poems.

Every man seeks automatically to heighten his feelings of pleasure and to relieve his feelings of pain. The artist is the man who finds that he can gain such enhancement or relief, not only by the direct action of giving expression to his feeling, but also by arousing a kindred feeling in others. Hence originates in him that desire to transmit his moods to an external audience, and there also arises the endeavour to give the artistic product a form which may facilitate the revival of the original state in an ever-widening circle of sympathisers.

"Beyond the fact that art has been obliged to avail itself of media which have originally been called into existence by utilitarian, non-æsthetic needs, there lies another fact. To these external 'origins' we can also trace some of the most important qualities which we appreciate in a work of art. In this way it is open to us to explain how several of the virtues of art, as we know it, may be derived from the primitive needs which it subserved; how, for instance, the lucidity of art may find its explanation in art's use for conveying information; how the sensuous and attractive qualities of all art may be traced to the need for propitiating favour; how the power that resides in art to trace and stimulate the mind may be transmitted from the days when the artist was appointed to nerve his fellows for work or war. And, lastly, it might be argued that a most characteristic quality of art—the imagination—which is in a sense faith in the reality of the unreal, may have been immensely heightened by the use of art for purposes of magic, which fuses the visible and the invisible."

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THE PARTIAL DIFFERENTIAL EQUATIONS OF MODERN MATHEMATICAL PHYSICS.

Die Partiellen Differentialgleichungen der mathematischen Physik. Nach Riemann's Vorlesungen. Fourth edition. Revised and rewritten by Heinrich Weber. Vol. i. Pp. xvii + 506. (Brunswick: Friedrich Vieweg und Sohn, 1900.)

THE lectures, delivered at the University of Göttingen by Prof. Bernard Riemann in the sessions of 1854-55, of 1860-61 and in the summer of 1862, have, thanks to the volume brought out after Riemann's death under the editorship of Karl Hattendorff, long ranked among the mathematical classics. The third and last edition of "*Partielle Differentialgleichungen*" appeared in 1882, and two years ago Prof. Heinrich Weber was entrusted with the task of bringing out a fourth

edition. There were three possible ways in which this task could have been fulfilled. One way was to republish the edition of 1882, with trifling additions and alterations. The second way was to retain the existing text, but to add copious notes together with references to recent developments bordering on the subject of Riemann's lectures. The third way was to write an entirely new book, based, indeed, on the earlier editions, but completely brought up to date by the embodiment of the new methods and problems that have come into existence in connection with discoveries in mathematics and physics extending over nearly twenty years from the date of the last edition, and nearly forty years from the time when the lectures were given by Riemann.

Prof. Weber has adopted the last of these alternatives, and by so doing has produced a treatise which will be invaluable to the modern mathematical physicist. How far the present treatise is to be regarded as a new work written by Prof. Weber may be inferred from the fact that this, the first volume only, covers 506 pages, as compared with a total of 325 in Hattendorff's edition, and all the last 350 pages are new.

The first part, dealing with analytical methods, corresponds more or less closely with the first three sections of Hattendorff's edition. It deals with definite integrals, infinite series and the differential equations of common occurrence in physics, especially linear equations with constant coefficients. In this portion we are indebted to Prof. Weber for an amplification of the treatment of Fourier's series and Fourier's double integral theorem, for a more precise treatment of continuity and for entirely new sections dealing with surface and volume integrals, functions of complex variables and conformal representation, and Bessel's functions, the last named addition occupying forty pages.

The second part is entirely new. In it Prof. Weber discusses linear infinitesimal deformations and then gives us a chapter on vectors, in which the modern notions of "curl" and "divergence" are fully explained, and expressions for the curl of a vector given in orthogonal coordinates. This is followed by sections on theory of the potential, including Green's theorem and potentials of ellipsoids. The next section deals with spherical harmonics, and this is followed by a short summary of the principles of dynamics, including the Hamiltonian equations and least action.

The only branches of physics treated in Hattendorff's edition were conduction of heat, elasticity (including vibrations) and hydrodynamics. The absence of any reference to electricity and magnetism is accounted for by the fact that these subjects, together with gravitation, were treated by Riemann in a separate course, of which an edition was also prepared for press by Hattendorff in 1876. The third part of the present volume forms a treatise on the mathematical theory of electricity and magnetism, for which Prof. Weber is thus solely responsible. The fundamental principles of electrostatics and magnetism are based on the hypothesis of a continuous medium, the electrical and magnetic properties of which depend on the existence at every point of space of certain vector quantities satisfying stated laws; and the subject is thus introduced much after the manner

adopted by Hertz. Among the problems depending for their solution upon the method of conformal representation, we notice an application of the transformation of Schwarz and Christoffel to the distribution of electricity on a prism, an example which practically amounts to an exposition of this transformation.

The subject of contact electricity, too, receives ample mathematical treatment. Perhaps, however, the most interesting sections are those dealing with electrolysis; and this interest is largely due to the important part which Prof. Weber himself has played in advancing our theories of this difficult subject. A comparison of these sections, in which the problem of electrolysis is made to depend on the solution of differential equations which Weber integrates in certain special cases, with the fragmentary information contained in text-books of forty years ago, is sufficient indication of the progress which has been made during the past half century in developing new fields of study in applied mathematics, and in co-ordinating and perfecting the mathematical treatment of electricity.

Steady flow of electricity, and the fundamental principles of "electrodynamics" (as it used to be and still sometimes is called), occur in their proper places in the present volume. No mention, however, is made of Hertzian oscillations, which are to be dealt with in the forthcoming second volume in connection with the theory of oscillations in general. The remaining subjects to be treated in the latter volume include conduction of heat, hydrodynamics and elasticity.

Mathematicians will, of course, not be satisfied with the present treatment of such matters as convergence of series and of integrals, and on the other hand physicists will require to supplement the volume with other works containing a fuller consideration of the experimental aspect of the various theories. It was no purpose of Prof. Weber's to aim at completeness in either of these respects. The object of the book is rather to furnish a statement of results both in pure mathematics and in physics, and to indicate the methods by which the former results, used in conjunction with the latter, lead to the mathematical solution of physical problems. As an illustration of the spirit of the book, we may notice the article on semi-convergent series, where the use of these series is explained mainly by the consideration of an illustrative example. Again, as Prof. Weber points out, there are many physical problems which can only be solved by approximate methods of little or no mathematical interest, and these again are omitted.

Now a book of this character appeals to a considerable class of present-day physicists. Forty years ago physical laboratories hardly existed, and the pioneers of physics in this country were Cambridge wranglers who approached the subject from its mathematical side exclusively. Now that physical laboratories are scattered all over the country, and that the working man can attend science classes close to his own door, we are running to the opposite extreme, and there is an ever-increasing class of student who requires to master the mathematics required for his physical studies, but who starts his mathematical reading too late in the day to work up step by step from the very beginning. As was pointed out by Riemann

in the introduction reproduced in Hattendorff's first edition, a science of physics (or more literally "a scientific physics") first existed after the discovery of the differential calculus. A sound knowledge of the differential and integral calculus is assumed in this book, but in Germany such a knowledge is acquired by the majority of students at the commencement of their academic curriculum, a stage where, in this country, many students are still attending lectures on fractions, highest common factor and Euclid. Those possessing the necessary preliminary training will find in Weber's new edition of Riemann an excellent introduction to the methods of applying mathematical principles to the problems of modern physics.

G. H. B.

THE CLASSIFICATION OF EARS.

The Human Ear, its Identification and Physiognomy.

By Miriam Anne Ellis. Pp. x+225. (London: A. and C. Black, 1900.) Price 3s. 6d. net.

A SIMPLE, workable, absolutely trustworthy system is still urgently wanted for the detection of criminals, and if the authoress of this book has succeeded she certainly deserves the thanks of all the Governments of Europe. Whatever worth her method may have when it comes to be applied practically, it has some decided drawbacks when the data are examined on which it is founded.

It so happened that about seven years ago the reviewer came to the conclusion that the external ear ought to yield some clue to the relationship of man and ape, and of one race of man to another. As is well known, the characters of the ear are fully inherited, and afford fairly trustworthy clues to family relationship, of which the authoress gives some good illustrations. Founding his method of observation and classification on data derived from a study of the development and comparative anatomy of the external ear, the reviewer proceeded to examine by hundreds the various peoples and races living on the shores of the North Sea, first on the Continental side, then on the British, to see how far the data he accumulated would support the semi-traditional accounts available concerning the early Saxon invasions of Britain. These observations were continued into the Highlands of Scotland, to Ireland and Wales. To test the "criminal-mark" theory of Lombroso and many others, he examined the ears of more than 800 confirmed criminals, and of more than two thousand inmates of asylums for the insane, situated in parts of the country where he had already examined the ears of the sane. Altogether the ears of more than 40,000 people of different races and of different moralities, besides those of about 300 apes and anthropoids, were examined, but the total results of this elaborate investigation were almost entirely of a negative nature.

The authoress appears to take it for granted, and evidently has not inquired into the matter, that the ear of the criminal is peculiar. If the reviewer's methods and observations are correct, the confirmed criminal's ear is the ear of the average inhabitant of Great Britain. Nor did the ears of the insane differ, on an average, from those of the people from which they were drawn, and if the authoress had carried her observations over a number

of men of genius or of high ability, instead of drawing elaborate deductions from single observations, she would probably have arrived at a similar conclusion as to them.

The great difficulty in a matter of this kind is to arrive at a method of classification, and it is in this that all the systems propounded break down when applied practically, and the system propounded here is worse than those that have gone before it. In her classification, the first division is a separation of ears into (1) large; (2) medium; (3) small. Unfortunately, she proposes no definite measurements, but if she did it would be found that a great proportion of ears fell on the limits of the medium line, and it would be a matter of the greatest difficulty to say to which of the great divisions it belonged. There is another great obstacle to the application of measurement of the ear to detection of criminals, of which the authoress is unaware. As Schwalbe showed years ago, and as the authoress may verify any quiet half hour during sermon time, the ear, in the later decades of life, undergoes a very considerable growth—enough to shift the ear of a woman aged forty from the medium division to the large division when she is aged sixty.

The authoress has used one of the most variable and untrustworthy features of the human ear for the purpose of subdividing and indexing the forms in which it is found. She detects in its helix (the upper and posterior border of the ear) five divisions, separated by indentations more or less marked. The three great groups of large, medium and small ears are subdivided according to which and how many of these divisions of the helix are present. In many cases no two observers would agree as to the number of helical subdivisions present, which is not remarkable when it is remembered that the helix on the posterior border is a vestigial structure, the result of the infolding of the free margin of the ear. The amount of infolding does not indicate, as the authoress supposes, certain psychological peculiarities, but merely the degree of retrogression in the ear examined. Like all truly vestigial structures, the infolded margin of the helix is subject to such a variety of forms that it defies classification.

One or two interesting, although minor, points might also be mentioned. The statement that the length of the ear depends on the length of the nose, and that the measurement of the one is identical with that of the other, will be found, on trial, to be the exception and not the invariable rule. In most anatomical works the relation of the breadth to the length of the ear is used as a method of classification; quite a useless one, in the reviewer's opinion. The statement made here is that "the width of the pinna should be at its middle part exactly half its length. . . . Any deviation from these exact measurements at once forms a valuable aid in identification." The scientific part of this book was read in the Anthropological Section at the meeting of the British Association at Bristol in 1898, and many of the observations it contains were made on the ears of eminent men of science. The authoress proposes the term of "otomorphology" to cover the science of the external ear, but from the phrenological character given it by this work perhaps the name of "earistry" were better.

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